VALIDITY OF THE PHASE SHIFT CALCULATION OF ELECTRON SCATTERING BY BRYSK METHOD

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For a central potential $V(r)$ the Born relation for the determination of phase shifts of elastic scattering is

$$\delta_l = -\frac{2km}{\hbar^2}\int_0^\infty r^2 j_l^2(kr)V(r)dr \qquad \ldots \quad (1)$$

where $k = \frac{(2mE)^{\frac{1}{2}}}{\hbar}$, E and m are the energy and the mass respectively of the incident particle. $j_l(kr)$ and $y_l(kr)$ are the spherical Bessel and Neumann functions respectively. Brysk (1962) has given an improved modification of equation (1) which is as follows

$$\tan\delta_l = \frac{-\frac{2km}{\hbar^2}\int_0^\infty r^2 j_l^2(kr)V(r)dr}{1-\frac{2km}{\hbar^2}\int_0^\infty r^2 j_l(kr)y_l(kr)V(r)dr} \qquad \ldots \quad (2)$$

Brysk after comparing the values of s-wave phase shift δ_0 obtained from the above relation and from exact calculation for the case of square well potential has found that equation (2) can be used to extend the calculation of s-wave phase shift to much lower energies where relation (1) fails totally and for higher energies relation (2) gives always better approximation than equation (1), at very high energies both the equations give the same exact result.

It may be worthwhile to investigate how the Brysk method fares for potentials other than the square well, for a screened coulomb potential as in the case of He atom, we have calculated δ_0 and δ_1 for incident energies at 5 e.v., 13.5 e.v. and 121.8 ev. and have compared our theoretical values with those obtained by Born relation and the exact values obtained by Mc. Dougall (1932) by numerical method.

For He atom the field given by Hartree has been used. The integrals involved in the Brysk formula have been evaluated numerically and the results are tabulated as follows.

TABLE

Energy E in ev.	δ_0 Present values	δ_0 Born values	δ_0 Exact values	δ_1 Present values	δ_1 Born values	δ_1 Exact values
5 ev.	2.329	.380	1.659	.025	.013	.020
13.5 ev.	2.01	.57	1.40	.094	.04	.07
121.8 ev.	1.21	.75	.90	.44	.24	.27

A comparison of our calculated values with the values obtained by using Born relation and the exact values shows that both for δ_0 and δ_1 the Brysk method is somewhat superior to Born-approximation at 5 ev. and 13.ev.; but it fails to give a satisfactory result at 121.8 ev. where Born approximation gives values nearer to exact ones than does Brysk one.

Brysk has pointed out that it is not possible to assign a priori the limits of validity of the method and the probable error. He has, however, mentioned that his method should hold fairly good for a short range potential. In our case of the screened coulomb field, the Brysk phases exceed the values obtained exactly, whereas the Born relation gives results which are always less than the exact values.

Because of the rather slow decrease of the integral occurring in the denominator of equation (2) the values of the phases as given by the Brysk method is considerably higher than the exact values even at the moderately high energy of 121.8 ev. It may however be remarked that at very high energy where the value of the integral approaches zero, this approximation is expected to give result quite similar to that obtained by Born relation.

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